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DEVELOPING ALGORITHMS OF OPTIMAL FORECASTING AND FILTERING FOR SOME CLASSES OF NONSTATIONARY RANDOM SEQUENCES

The problem of forecasting and filtering non-stationary random sequences is solved in the article. Optimal forecasting and filtering are performed using linear estimates and minimizing the mean squared error. For non-stationary random sequences, even with the correlation functions of the simplest form, such studies were not conducted. In this work, on the examples of non-stationary sequences, the problem of forecasting and filtering is solved explicitly. The correlation function image is obtained using the Hilbert approach, which allows one to calculate correlation functions as scalar products in a corresponding Hilbert space. The solution of the extrapolation problem with particular correlation function considered in the article can be used to simulate filtration and forecasting processes in real systems in the case of non-stationary random signals.

Key words: correlation function, mathematical expectation, forecasting and filtering of nonstationary random sequences and processes, mean square error.

Н. В. ЧЕРЕМСЬКА

ПОБУДОВА АЛГОРИТМІВ ОПТИМАЛЬНОГО ПРОГНОЗУ І ФІЛЬТРАЦІЇ ДЛЯ ДЕЯКИХ КЛАСІВ НЕСТАЦІОНАРНИХ ВИПАДКОВИХ ПОСЛІДОВНОСТЕЙ

Розв’язується задача прогнозу і фільтрації нестаціонарних випадкових послідовностей. Оптимальні прогноз і фільтрація здійснюються за допомогою лінійних оцінок та мінімізації середньої квадратичної помилки. Для нестаціонарних випадкових послідовностей, навіть з кореляційними функціями найпростішого вигляду, такі дослідження не проводились. У цій роботі на прикладах нестаціонарних послідовностей задача прогнозу та фільтрації вирішується явно. Для отримання зображень кореляційних функцій використовується гільбертів підхід, який дозволяє обчислювати кореляційні функції як скалярні добутки у відповідному гільбертовому просторі. Розв’язок екстраполяційної задачі з частковими видами кореляційної функції, який було розглянуто в статті, може бути використаний для моделювання процесів фільтрації та прогнозу в реальних системах у випадку нестаціонарних випадкових сигналів.

Ключові слова: кореляційна функція, математичне очікування, прогноз та фільтрація нестаціонарних випадкових послідовностей і процесів, середня квадратична помилка.

Н. В. ЧЕРЕМСКАЯ

ПОСТРОЕНИЕ АЛГОРИТМОВ ОПТИМАЛЬНОГО ПРОГНОЗА И ФИЛЬТРАЦИИ ДЛЯ НЕКОТОРЫХ КЛАССОВ НЕСТАЦИОНАРНЫХ СЛУЧАЙНЫХ ПОСЛЕДОВАТЕЛЬНОСТЕЙ

Решается задача прогноза и фильтрации нестационарных случайных последовательностей. Оптимальные прогноз и фильтрация осуществляются с помощью линейных оценок и минимизации средней квадратичной ошибки. Для нестационарных случайных последовательностей, даже с корреляционными функциями простейшего вида, такие исследования не проводились. В этой работе на примерах нестационарных последовательностей задача прогноза и фильтрации решается явно. Для получения представлений корреляционных функций используется гильбертов поход, позволяющий вычислять корреляционные функции как скалярные произведения в соответствующем гильбертовом пространстве. Решение экстраполяционной задачи с частными видами корреляционной функции, рассмотренное в статье может быть использовано для моделирования процессов фильтрации и прогноза в реальных системах в случае нестационарных случайных сигналов.

Ключевые слова: корреляционная функция, математическое ожидание, прогноз и фильтрация нестационарных случайных последовательностей и процессов, средняя квадратическая ошибка.

Introduction. The tasks of predicting the values of random processes (sequences) for known values in the past or the allocation of a signal in the background of random noise are partial but very important problems of the general theory of linear transformations of a random signal. Solving the extrapolation problem with particular correlation function, which is calculated for various cases of the spectrum of a non-selfadjoint bounded operator, can be used to simulate the filtration and prediction processes in real systems in the case of non-stationary random signals.

Analysis of recent research. A large quantity of works [1, 2, 4, 8 – 11] are devoted to forecasting and filtering of non-stationary random sequences. In the main, these papers consider the prognosis and filtering of stationary random sequences on the basis of the theory of functions of a complex variable and some classes of functional spaces, or by the approach *proposed by Kalman*, which leads to a rather complicated recurrence procedure. Construction of the optimal filter on the finite number of values of random sequence encounters significant difficulties associated with the need to calculate explicit determinants of the n-th order of a special form. Therefore, the complexity of such calculations and the cumbersome nature of explicit formulas did not contribute to a significant advance in solving this problem. An exception is the work [8], where some explicit extrapolation formulas are obtained in the case of a stationary random sequence with a correlation function of the form:

$$B(k) = M\xi(n+k)\overline{\xi(n)} = \begin{cases} \dfrac{m-k}{m}, & |k| \le m; \\ 0, & |k| > m. \end{cases}$$

Formulation of the problem. The article is based on simpler estimates of random functions in the future moment of time, linear with respect to the values of the prehistory of processes. For non-stationary random sequences, even with the correlation functions of the simplest form, such studies were not conducted. This circumstance is explained, in particular, by the lack of meaningful examples of truly significant correlation functions that describe random processes with complex, unlike stationary, spectra. The studies carried out in [5, 6] allow us to bridge this gap and construct simple prognostic algorithms for non-stationary random functions.

Mathematical model. Consider a random sequence $z(n)$ with mathematical expectation $Mz(n) = 0$ and a known correlation function $K_{zz}(n,m)$. Suppose at times $p_1, p_2, ..., p_{n_1}$ the known values $u(p_k)$ $(k = 1, ..., n_1)$ are determined by the expression:

$$u(p_k) = z(p_k) + \xi(p_k), \tag{1}$$

where $\xi(n)$ is discrete white noise with $M\xi(n) = 0$ and a correlation function $K_{\xi\xi}(m, n) = s_0\delta_{nm}$.

Let's look for a linear optimal estimate $z(n),\ n = 1, ..., n_1$, such that $\hat{z}(n)$: $\hat{z}(n) = \sum_{p=1}^{n_1} G(n, p)u(p)$. Obviously, it is unbalanced, and its efficiency is ensured by the minimum of the mean square error:

$$\sigma^2(G) = M\left|z(n) - \hat{z}(n)\right|^2. \tag{2}$$

The standard procedure for finding the minimum of $\sigma^2(G)$ leads to the following system of algebraic equations for $G_0(n,q)$ which provide $\min \sigma^2(G)$:

$$\sum_{q=1}^{n_1} G_0(n, q)K_{uu}(q, p) = K_{zu}(n, p). \tag{3}$$

In the case when $z(n)$ is a stationary sequence, $G_0(n, q)$ can be considered a function that depends on the difference between the discrete arguments and, if $z(n)$ is uncorrelated with $\xi(n)$ or permanently connected with it, then equation (3) becomes the equation:

$$\sum_{m=n}^{n+n_1} G_0(m)K_{uu}(n-p-m) = K_{zu}(n-p), \tag{4}$$

which was considered in [1, 8, 9].

Let's turn to the consideration of non-stationary sequence $z(n)$ of the form $z(n) = \lambda_0^n z_0(\omega)$ $\left(\lambda_0 = \overline{\lambda_0} \ne \pm 1\right)$ with a *Hankel correlation function* [6, 7] $K_{zz}(n, m) = \lambda_0^{n+m} M\left|z_0(\omega)\right|^2$.

We will show that in this case the extrapolation problem is solved explicitly.

System (3) in this case takes the form:

$$\sum_{q=1}^{n_1} G_0(n,q)[\lambda_0^{q+p} M\left|z_0(\omega)\right|^2 + s_0\delta_{nm}] = \lambda_0^{n+p} M\left|z_0(\omega)\right|^2,$$

or

$$s_0 G(n,q)+(\sum_{q=1}^{n_1} G_0(n,q)\lambda_0^q)\lambda_0^p M\left|z_0(\omega)\right|^2=\lambda_0^{n+p} M\left|z_0(\omega)\right|^2 .$$

We are looking for the solution $G_0(n,q)$ of this problem such that $G_0(n,q)=\lambda_0^n L(p)$:

$$s_0 L(p)+\sum_{q=1}^{n_1} L(q)\lambda_0^{q+p} M\left|z_0(\omega)\right|^2=\lambda_0^p M\left|z_0(\omega)\right|^2, \tag{5}$$

where $p=0, ..., n_1$.

Setting $\sum_{q=1}^{n_1} L(q)\lambda_0^q \equiv X$, (5) can be written as

$$s_0 L(p)=\lambda_0^p M\left|z_0(\omega)\right|^2-\lambda_0^p X M\left|z_0(\omega)\right|^2 . \tag{6}$$

Multiplying both sides of equation (6) by λ_0^p and summing in p, we get $s_0 X+DX=D$, where

$$D=\sum_{p=1}^{n_1}\lambda_0^{2p} M\left|z_0(\omega)\right|^2=\frac{\lambda_0^{2n_1+2}-\lambda_0^2}{\lambda_0^2-1} M\left|z_0(\omega)\right|^2 .$$

Thus,

$$X=\frac{D}{s_0 I+D}=\frac{\left(\lambda_0^{2n_1+2}-\lambda_0^2\right) M\left|z_0(\omega)\right|^2}{s_0\left(\lambda_0^2-1\right)+\left(\lambda_0^{2n_1+2}-\lambda_0^2\right) M\left|z_0(\omega)\right|^2} . \tag{7}$$

From equation (6) we obtain $L(p)=\frac{1}{s_0}(I-X)\lambda_0^p M\left|z_0(\omega)\right|^2$, where $p=1, ..., n_1$.

Hence,

$$G_0(n,p)=\lambda_0^{n+p}\frac{1}{s_0}(I-X) M\left|z_0(\omega)\right|^2 .$$

Thus, the unbalanced effective estimate takes the form:

$$\hat{z}(n)=\lambda_0^n\frac{1}{s_0}\left(I-\frac{\left(\lambda_0^{2n_1+2}-\lambda_0^2\right) M\left|z_0(\omega)\right|^2}{s_0\left(\lambda_0^2-1\right)+\left(\lambda_0^{2n_1+2}-\lambda_0^2\right) M\left|z_0(\omega)\right|^2}\right) M\left|z_0(\omega)\right|^2\sum_{p=1}^{n_1}\lambda_0^p u(p) . \tag{8}$$

In this case, the mean square error becomes:

$$\sigma^2(G_0)=M\left|z(n)-\hat{z}(n)\right|^2=K_{zz}(n,n)-K_{z\hat{z}}(n,n)-K_{\hat{z}z}(n,n)+K_{\hat{z}\hat{z}}(n,n)=$$

$$=\lambda_0^{2n} M\left|z_0(\omega)\right|^2-2\frac{\lambda_0^n}{s_0}(I-X) M\left|z_0(\omega)\right|^2\sum_{p=1}^{n_1}\lambda_0^p\lambda_0^{n+p} M\left|z_0(\omega)\right|^2+$$

$$+\left(\frac{\lambda_0^n}{s_0}(I-X) M\left|z_0(\omega)\right|^2\right)^2\sum_{p,q=1}^{n_1}\lambda_0^{p+q}(\lambda_0^{p+q} M\left|z_0(\omega)\right|^2+s_0\delta_{pq})=$$

$$=\lambda_0^{2n} M\left|z_0(\omega)\right|^2-2\frac{\lambda_0^{2n}}{s_0}(I-X) Y\left(M\left|z_0(\omega)\right|^2\right)^2+\left(\frac{\lambda_0^n}{s_0}(I-X) M\left|z_0(\omega)\right|^2\right)^2\left(Y^2 M\left|z_0(\omega)\right|^2+s_0 Y\right),$$

where $Y=\frac{\lambda_0^{2n_1+2}-\lambda_0^2}{\lambda_0^2-1}$.

Let us now consider a more interesting case of non-stationary sequence, which after immersion in the Hilbert space [1, 7] is presented by a sequence of the form $z_n=f(n)=A^n z_0$, where A is a non-selfadjoint bounded operator.

Let $H_2=L^2_{[0;1]}$, and the operator A have the form:

$$Af(x)=\lambda_0 f(x)+i\int_0^1\varphi(x)\overline{\theta(x)} f(y)\,dy,\ \lambda_0\neq\overline{\lambda_0},\ \left(\dim\overline{\operatorname{Im} AL^2_{[0;1]}}=\infty\right).$$

That is, consider the one-dimensional perturbation of the multiplication operator by a complex constant.

The choice of such an operator is also due to the fact that the theory of non-selfadjoint operators did not actually concern the *triangular* and *Fredholm operators* of $\dim\overline{\operatorname{Im} AH}=\infty$ [12].

$$(A)^n f = f(n),\ f(n+1) = Af(n),\ f(n)\big|_{n=0} = f_0,$$

$$f(n+1, x) = \lambda_0 f(n, x) + i\int_0^1 \varphi(x)\overline{\theta(x)} f(n, y)\,dy,$$

$$f(n+1, x) = \lambda_0 f(n, x) + i\varphi(x)\alpha_n, \tag{9}$$

where $\alpha_n = \int_0^1 \overline{\theta(x)} u(n, y)\,dy$, $f(k, x) = \lambda_0^k f(0, x) + i\varphi(x)\sum_{j=0}^{n-1} \alpha_j \lambda_0^{k-j-1}$.

To find α_j, we multiply (9) by $\bar{\theta}(x)$ and integrate, then we obtain the recurrence ratio for α_j:

$$\alpha_{n+1} = \lambda_0\alpha_n + i\gamma\alpha_n,\ \alpha_n\big|_{n=0} = \alpha_0 = \int_0^1 f_0(x)\overline{\theta(x)}\,dx,$$

where $\gamma = \int_0^1 \varphi(x)\overline{\theta(x)}\,dx$. Thus, $\alpha_n = (\lambda_0 + i\gamma)^n \alpha_0$. Hence, $f(n, x) = \lambda_0^n f_0(x) + \varphi(x)\alpha_0 \dfrac{(\lambda_0 + i\gamma)^n - \lambda_0^n}{\gamma}$. Consequently, $A^n f = \lambda_0^n f_0^{(1)}(x) + (\lambda_0 + i\gamma)^n f_1^{(1)}(x)$, where $f_0^{(1)}(x) = f_0(x) - \dfrac{\varphi(x)\alpha_0}{\gamma}$, $f_1^{(1)}(x) = \dfrac{\varphi(x)\alpha_0}{\gamma}$.

In this case, the correlation function has the form:

$$K(n, m) = \langle f(n),\ f(m)\rangle = \langle A^n f_0(x),\ A^m f_0(x)\rangle = \lambda_0^n \bar{\lambda}_0^m \left\| f_0^{(1)}(x)\right\|^2 + (\lambda_0 + i\gamma)^n (\bar{\lambda}_0 - i\bar{\gamma})^m \left\| f_1^{(1)}(x)\right\|^2 +$$
$$+\lambda_0^n (\bar{\lambda}_0 - i\bar{\gamma})^m \langle f_0^{(1)}(x),\ f_1^{(1)}(x)\rangle + (\lambda_0 + i\gamma)^n \bar{\lambda}_0^m \langle f_1^{(1)}(x),\ f_0^{(1)}(x)\rangle.$$

The relation $K_{uu}(n, m) = K_{zz}(n, m) + K_{\xi\xi}(n, m)$ implies that

$$s_0 G_0(n, q) + \sum_{q=1}^{n_1} G_0(n, q)[\lambda_0^q \bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + (\lambda_0 + i\gamma)^q (\bar{\lambda}_0 - i\bar{\gamma})^p \left\| f_1^{(1)}(x)\right\|^2 +$$
$$+\lambda_0^q (\bar{\lambda}_0 - i\bar{\gamma})^p \langle f_0^{(1)}(x), f_1^{(1)}(x)\rangle + (\lambda_0 + i\gamma)^q \bar{\lambda}_0^p \langle f_1^{(1)}(x), f_0^{(1)}(x)\rangle] =$$
$$= \lambda_0^n \bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + (\lambda_0 + i\gamma)^n (\bar{\lambda}_0 - i\bar{\gamma})^p \left\| f_1^{(1)}(x)\right\|^2 +$$
$$+\lambda_0^n (\bar{\lambda}_0 - i\bar{\gamma})^p \langle f_0^{(1)}(x),\ f_1^{(1)}(x)\rangle + (\lambda_0 + i\gamma)^n \bar{\lambda}_0^p \langle f_1^{(1)}(x),\ f_0^{(1)}(x)\rangle. \tag{10}$$

We shall show that in the case under consideration, the solution of the system can be determined in explicit form.

Since the right-hand side is the sum of four terms, the solution of (10) is sought as a sum of four terms. Let $G_0 = G_1 + G_2 + G_3 + G_4$, where G_1 is the solution of the following system

$$s_0 G_0(n, q) + \sum_{q=1}^{n_1} G_0(n, q)[\lambda_0^q \bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + (\lambda_0 + i\gamma)^q (\bar{\lambda}_0 - i\bar{\gamma})^p \left\| f_0^{(1)}(x)\right\|^2 + \lambda_0^q (\bar{\lambda}_0 - i\bar{\gamma})^p \langle f_0^{(1)}(x),\ f_1^{(1)}(x)\rangle +$$
$$+(\lambda_0 + i\gamma)^q \bar{\lambda}_0^p \langle f_1^{(1)}(x),\ f_0^{(1)}(x)\rangle] = \lambda_0^n \bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2, \tag{11}$$

The supplements G_2, G_3, G_4 satisfy similar equations.

The solution of (11) admits the representation: $G_1(n, p) = \lambda_0^n F_1(p)\left\| f_0^{(1)}(x)\right\|^2$. Hence, we receive:

$$s_0 F_1(p) + \sum_{q=1}^{n_1} F_1(q)[\lambda_0^q \bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + (\lambda_0 + i\gamma)^q (\bar{\lambda}_0 - i\bar{\gamma})^p \left\| f_1^{(1)}(x)\right\|^2 +$$
$$+\lambda_0^q (\bar{\lambda}_0 - i\bar{\gamma})^p \langle f_0^{(1)}(x), f_1^{(1)}(x)\rangle + (\lambda_0 + i\gamma)^q \bar{\lambda}_0^p \langle f_1^{(1)}(x), f_0^{(1)}(x)\rangle] = \bar{\lambda}_0^p;$$

$$s_0 F_1(p) + \left(\sum_{q=1}^{n_1} F_1(q)\lambda_0^q\right)\bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + \left(\sum_{q=1}^{n_1} F_1(q)(\lambda_0 + i\gamma)^q\right)(\bar{\lambda}_0 - i\bar{\gamma})^p \times$$
$$\left\| f_1^{(1)}(x)\right\|^2 + \left(\sum_{q=1}^{n_1} F_1(q)\lambda_0^q\right)(\bar{\lambda}_0 - i\bar{\gamma})^p \langle f_0^{(1)}(x),\ f_1^{(1)}(x)\rangle +$$

$$+\left(\sum_{q=1}^{n_1} F_1(q)(\lambda_0+i\gamma)^q\right)\bar{\lambda}_0^p \left\langle f_1^{(1)}(x),\, f_0^{(1)}(x)\right\rangle = \bar{\lambda}_0^p .$$

Let's introduce the notations: $C_N^{(1)} = \sum_{q=1}^{n_1} F_1(q)\lambda_0^q,\ C_N^{(2)} = \sum_{q=1}^{n_1} F_1(q)(\lambda_0+i\gamma)^q$. In these notations, the previous equation takes the form:

$$s_0 F_1(p) + C_N^{(1)}\left(\bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + \left(\bar{\lambda}_0 - i\bar{\gamma}\right)^p \left\langle f_0^{(1)}(x),\, f_1^{(1)}(x)\right\rangle\right) +$$

$$+C_N^{(2)}\left(\left(\bar{\lambda}_0 - i\bar{\gamma}\right)^p \left\| f_1^{(1)}(x)\right\|^2 + \bar{\lambda}_0^p \left\langle f_1^{(1)}(x),\, f_0^{(1)}(x)\right\rangle\right) = \bar{\lambda}_0^p . \tag{12}$$

We multiply (12) by λ_0^q and sum up to obtain

$$s_0 C_N^{(1)} + C_N^{(1)} A_N + C_N^{(2)} B_N = F_N ,$$

where

$$A_N = \sum_{q=1}^{N} \lambda_0^q \left(\bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + \left(\bar{\lambda}_0 - i\bar{\gamma}\right)^p \left\langle f_0^{(1)}(x),\, f_1^{(1)}(x)\right\rangle\right);$$

$$B_N = \sum_{q=1}^{N} \lambda_0^q \left(\left(\bar{\lambda}_0 - i\bar{\gamma}\right)^p \left\| f_1^{(1)}(x)\right\|^2 + \bar{\lambda}_0^p \left\langle f_1^{(1)}(x),\, f_0^{(1)}(x)\right\rangle\right), \quad F_N = \sum_{q=1}^{N} \lambda_0^q s_0 F_1(p) .$$

We multiply (12) by $(\lambda_0+i\gamma)^q$ and sum up to get

$$s_0 C_N^{(2)} + C_N^{(2)} D_N + C_N^{(2)} L_N = H_N ,$$

where

$$D_N = \sum_{q=1}^{N} (\lambda_0+i\gamma)^q \left(\bar{\lambda}_0^p \left\| f_0^{(1)}(x)\right\|^2 + \left(\bar{\lambda}_0 - i\bar{\gamma}\right)^p \left\langle f_0^{(1)}(x),\, f_1^{(1)}(x)\right\rangle\right);$$

$$L_N = \sum_{q=1}^{N} (\lambda_0+i\gamma)^q \left(\left(\bar{\lambda}_0 - i\bar{\gamma}\right)^p \left\| f_1^{(1)}(x)\right\|^2 + \bar{\lambda}_0^p \left\langle f_1^{(1)}(x),\, f_0^{(1)}(x)\right\rangle\right), \quad H_N = \sum_{q=1}^{N} (\lambda_0+i\gamma)^q s_0 F_1(p).$$

Consequently, we obtain a system of two linear equations with two unknowns:

$$\begin{cases} s_0 C_N^{(1)} + C_N^{(1)} A_N + C_N^{(2)} B_N = F_N , \\ s_0 C_N^{(2)} + C_N^{(2)} D_N + C_N^{(2)} L_N = H_N . \end{cases}$$

Thus, the solution of system (10) reduces to the solution of systems of two linear equations with two unknowns and can be found explicitly. Random sequences of the simplest form $z(n) = \lambda_0^n z_0(\omega)$, where λ_0 is in general a complex number, are widely used in applications, in particular, in the theory of pulse systems [3]. This is explained by the fact that if we consider the real and imaginary parts of the image $z(n) = x(n) + iy(n),\ \lambda_0 = \alpha_0 + i\beta_0$, then we obtain the following difference equation for $x(n)$:

$$x(n+2) - 2\alpha_0 x(n+1) + \left(\alpha_0^2 + \beta_0^2\right) x(n) = 0 ,$$

with random initial conditions

$$x(n)\big|_{n=0} = x_0(\omega),\quad x(n)\big|_{n=1} = \alpha_0 x_0(\omega) - \beta_0 y_0(\omega),$$

where $\lambda_0 = \alpha_0 + i\beta_0,\ z_0(\omega) = x_0(\omega) + iy_0(\omega)$, (for $y(n)$ the situation is similar).

Let us turn to algorithms for predicting random non-stationary processes and sequences.

Consider the forecast by the last value $\hat{\xi}(n+\theta),\ \theta > 0$ for a random sequence ($\hat{\xi}(t+\theta)$ of a random process), i.e. a zero order extrapolation. In this case, the forecast error is equal to: $\hat{\xi}(n+\theta) = \xi(n)$, and the forecast error e is equal to $e(n+\theta) = \xi(n+\theta) - \hat{\xi}(n+\theta) = \xi(n+\theta) - \xi(n)$. The average squared error σ^2 in this case has the form:

$$\sigma^2(n,\theta) = Me^2 = K(n+\theta, n+\theta) - 2K(n+\theta, n) + K(n, n), \tag{13}$$

where $K(n, m)$ is the random sequence correlation function.

Formula (13) in the particular case of non-stationary random processes becomes the known formula [13]:

$$\sigma^2(\theta) = Me^2 = K(0) - 2K(\theta) + K(0) = 2\left(K(0) - K(\theta)\right), \tag{14}$$

besides, when $\theta = 0$ $\sigma^2(\theta) = 0$, at $\theta = \infty$ $\sigma^2(\theta) = 2K(0)$ (assuming $\lim_{\theta\to\infty} K(\theta) = 0$).

In the general case, from (13) it follows that when $\theta = 0$, as in the stationary case, $\sigma^2(n,\theta) = 0$, and

$$\lim_{\theta\to\infty} K(n+\theta, n+\theta) = 0, \qquad (15)$$

then $\lim_{\theta\to\infty} \sigma^2(n,\theta) = K(n,n)$, hence, $\lim_{\substack{\theta\to\infty \\ n\to\infty}} \sigma^2(n,\theta) = 0$.

This distinguishes essentially the non-stationary case from the stationary one assuming (15) holds. (Note that (15) implies $\lim_{\theta\to\infty} K(n+\theta, n) = 0$ due to the known formula $|K(n,m)|^2 \le K(n,n)K(m,m)$.)

For non-stationary random processes $\xi(t)$ ($t-$continuous parameter) we get the expression for the average square of the last value forecast error similar to (13):

$$\sigma^2(t,\theta) = K(t+\theta, t+\theta) - 2K(t+\theta, t) + K(t,t).$$

Prospects for further research. The solution of the extrapolation problem with particular correlation function considered in the article can be used to simulate filtration and forecasting processes in real systems in the case of non-stationary random signals. The algorithm for finding the optimal estimate in form (8) is easy for hardware implementation. In relation to equation (10) we can say that the construction of an appropriate algorithm for finding an optimal mean square estimate does not cause significant difficulties.

Conclusions. The obtained algorithm for finding the optimal estimate of random non-stationary processes and sequences can be used for the analysis of statistically non-stationary signals, which is promising when solving many applied problems for which the non-stationary data are significant.

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